## Fundamental Concepts

January 2012

## Topics

Navigation and Ancillary Information Facility

- Preface
- Time
- Reference Frames
- Coordinate Systems
- Positions and States
- Aberration Corrections


## Preface

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- This tutorial introduces terminology and concepts used in the later SPICE tutorials.
- Some of this material is more difficult than what follows in later presentations.
- A complete understanding of this material is not essential in order to use SPICE.
- Still, we think this information may be helpful, so... on we go!


## Time

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- An epoch is an instant in time specified by some singular event
- Passage of a star across your zenith meridian
- Eclipse of a spacecraft signal as it passes behind a solid body
- Clocks
- Clocks count epochs specified by events such as: "regular" oscillations of a pendulum, quartz crystal, or electromagnetic radiation from a specified source, measured from an agreed upon reference epoch.
- Careful specification of epochs using clocks requires reference to the particular clock and the location of that clock.
- Time Systems
- Are agreed upon standards for "naming" epochs, measuring time, and synchronizing clocks


## Atomic Time and UTC

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- International Atomic Time (TAI)
- Statistical time scale
» Based on data from ~200 atomic clocks in over 50 national laboratories
- Maintained by Bureau International des Poids et Mesures (BIPM)
- Unit is the SI (System International) second
» duration of 9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium 133 atom
- TAI is expressed as a count of atomic seconds past the astronomically determined instant of midnight 1 Jan 1958 00:00:00
- Coordinated Universal Time (UTC)
- Civil Time at Greenwich England (~GMT)
- Usual Calendar Formats plus Hour:Minute:Second.fraction
- Relationship between TAI and UTC
- UTC + 10 seconds + number of leap seconds = TAI
» Valid only after Jan 01, 1972


## Astronomical Time - UT1

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- Astronomical Time (UT1) is an hour representation of the angle between the Greenwich zenith meridian and the location of the "computed mean sun."
- Used prior to atomic time for civil time keeping


## Tying UTC to Earth's Rotation (UT1)

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Ideally, UTC noon and astronomical noon at Greenwich (UT1) should occur simultaneously. However, the earth's rotation is not uniform. Eventually, UTC noon and astronomical noon at Greenwich get out of sync.


When the mismatch becomes greater than 0.9 atomic seconds, a "leap second" is added to (or removed from) the end of a designated UTC day-normally either June 30 or December 31.

The variations in the earth's rotation that cause leap seconds to be needed are not predictable.

## Leapseconds (+ and -)

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- "Normal" sequence of UTC time tags
- 1998 Dec 31 23:59:58.0
- 1998 Dec 31 23:59:59.0
- 1999 Jan 01 00:00:00.0
- 1999 Jan 01 00:00:01.0

Leap seconds complicate the task of finding the duration between two UTC epochs:

- You need to know when past leap seconds occurred to compute durations defined by pairs of past UTC epochs.
- Durations defined by pairs of future UTC epochs are indeterminate if leap seconds could occur in the interim.
- Sequence with a Positive Leapsecond
- 1998 Dec 31 23:59:58.0
- 1998 Dec 31 23:59:59.0
- 1998 Dec 31 23:59:60.0
- 1999 Jan 01 00:00:00.0
- 1999 Jan 01 00:00:01.0
- Sequence with a Negative Leapsecond
- 1998 Dec 31 23:59:57.0
- 1998 Dec 31 23:59:58.0
- 1999 Jan 01 00:00:00.0
- 1999 Jan 01 00:00:01.0


## Barycentric Dynamical Time

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- Barycentric Dynamical Time (TDB) and Ephemeris Time (ET) are synonyms in SPICE documentation.
- TDB is
- a mathematical Ideal used in the equations of motion.
- used as the independent time variable for many SPICE subroutine interfaces.
- related to ideal time at the Solar System Barycenter (TCB) by a scale factor, so that TDB advances on average at the same rate as TAI.


## Terrestrial Dynamical Time

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- Terrestrial Dynamical Time (TDT)
- TDT is the Ideal Time (proper time) on Earth at sea level
- TDT = TAI + 32.184 seconds
- The IAU has adopted the name "Terrestrial Time" (TT)
» But this is called TDT throughout SPICE documentation
- TDB and TDT have the same reference epoch (approximately 1 Jan 2000, 12:00:00 at Greenwich England)
- TDB and TDT advance at different rates.
- Variations are small: ~ 1.6 milliseconds
- Variations are periodic with a period of 1 sidereal year (to first order)
- Variations are due to relativistic effects

$$
\text { » TDB = TDT + } 0.001657 \sin (E+0.01671 \sin (E))
$$

- Use of TDT in the SPICE system is quite limited.
- SCLK kernels
- Duration computations involving UTC


## Spacecraft Clocks

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- Spacecraft have onboard clocks to control scheduling of observations, maneuvers, attitude adjustments, etc.
- Used to time stamp data
- Fundamental unit of time is the "tick"
- Smallest increment possible for spacecraft clock
- Spacecraft clock time is a count of ticks since some reference tick.
- The duration of the tick drifts with respect to other time systems because spacecraft clocks are not very stable


## More about Spacecraft Clocks

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- SCLK string formats vary from one spacecraft clock to the next.
- Cassini: Maximum reading for partition $1=1 / 4294967295.255$
» Partition number:1
» Seconds: 4294967295
» Ticks (for Cassini, unit = 1/256 second):
255
- Galileo: Maximum reading for partition 1 = 1/16777215:90:09:07
» Partition number:1
» "RIM" count (unit = $602 / 3$ seconds): 16777215
» "Mod 91" count (unit = 2/3 second): 90
» "RTI" count (unit = $1 / 15$ second): 9
" "Mod 8" count (unit = $1 / 120$ second): 7
- The format of spacecraft clock and the relationship between tick count and other time systems (usually UTC) is captured in a SPICE SCLK kernel


## SPICE Definitions:

## Reference Frames \& Coordinate Systems

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- A reference frame is an ordered set of three mutually orthogonal (possibly time dependent) unitlength direction vectors, coupled with a location called the frame's "center" or "origin."
- SPICE documentation frequently uses the shorthand "frame" instead of "reference frame."
- The ordered set of axes of a reference frame is also called a "basis."
- A coordinate system specifies the method of locating a point within a reference frame.



## Reference Frame Center

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- A reference frame's center is an ephemeris object whose location is coincident with the origin $(0,0,0)$ of the frame.
- The center of the IAU_<body> frame is <body>.
- The center of any inertial frame is (in SPICE) the solar system barycenter.
» True even for frames naturally associated with accelerated bodies, such as MARSIAU.
- A frame's center plays little role in specification of states
- Origin cancels out when doing vector arithmetic
» Whether positions of objects $A$ and $B$ are specified relative to centers C1 or C2 makes no difference:

$$
(A-C 1)-(B-C 1)=(A-C 2)-(B-C 2)=A-B
$$

- But the center *is* used in computing light time to centers of non-inertial frames
» When the aberration-corrected state of Titan as seen from the Cassini orbiter is computed in the body-fixed IAU_Titan frame, light time is computed from Titan's center to the Cassini orbiter, and this light time is used to correct both the state and orientation of Titan.


## Types of Reference Frames

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- Inertial
- Non-rotating
» With respect to fixed stars
- Non-accelerating origin
" Velocity is typically non-zero; acceleration is negligible
- Examples:
» J2000 (also called ICRF), B1950
- Non-Inertial
- Examples
" Body-fixed
- Centered at body center
- Topocentric
» Instrument
" Dynamic frames
- For example, frames defined by time-dependent vectors


## Frames Defined by Dynamics

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- Mean Equator
- Model gives mean direction of north pole of earth accounting for precession
- Defines $z$-axis of frame
- Defines a mean plane of equator
- Mean Ecliptic
- Model gives mean direction of the "pole" of the earth's orbit
- Defines a mean plane of the ecliptic
- Intersection of planes at a particular epoch
 determines x -axis



## J2000 (ICRF) Frame



## Rotating Frames

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- Rotating frames rotate with respect to Inertial Frames. Directions of axes are not constant w.r.t. inertial frames
- Centers may accelerate
- Examples:
- Body-fixed frames are tied to the surface of a body and rotate with it.
- Spacecraft-fixed frames are defined by the time-varying orientation of a spacecraft



## IAU Body-fixed Frames

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- Defined with simple models for position of spin axis and motion of prime meridian
- Z-axis points to the "north" side of the invariable plane of the solar system
- Invariable plane is perpendicular to the angular momentum vector of the solar system


## Invariable Plane of the solar system

## Topocentric Frames

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- Topocentric frames are attached to a surface
- Z-axis is parallel to the gravity gradient or orthogonal to reference spheroid
z points "up"


## Position Vector



One example of a topocentric frame. There are other types of topocentric frames: for example, the $z$-axis could point down, the $x$ axis North, and the $y$-axis East.

## Spacecraft and Instrument Frames

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- Defined relative to structures
- Spacecraft
- Scan platform
- Instrument
» For example you might have:
- z-axis lies along instrument boresight
- $x$ and $y$ axes defined by instrument characteristics


## State Vectors

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- The state of an object is its position and velocity relative to a second object
- In SPICE, these objects are often referred to as "target" and "observer" or "center"
- E.g. Saturn relative to Saturn barycenter; Titan relative to Huygens probe
- In the SPK subsystem a state is a six dimensional vector
- First three components are Cartesian position: $x, y, z$
- Second three components are Cartesian velocity: $d x / d t, d y / d t, d z / d t$
- Units are km, km/sec
- A state is specified relative to a reference frame


## Transforming States

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- To perform algebraic operations on states they must be in the same reference frame.
- Position-only frame transformations require only a rotation* matrix given as a function of time.

$$
\text { » } \mathrm{P}_{\mathrm{B}}(\boldsymbol{t})=\mathrm{R}_{\mathrm{A} \text { to } \mathrm{B}}(\boldsymbol{t}) \mathrm{P}_{\mathrm{A}}(t)
$$

- Position and velocity frame transformations require that we differentiate the above equation
" $d \mathrm{P}_{\mathrm{B}}(t) / d t=d \mathrm{R}_{\mathrm{A} \text { to } \mathrm{B}}(t) / d t \mathrm{P}_{\mathrm{A}}(t)+\mathrm{R}_{\mathrm{A} \text { to } \mathrm{B}}(t) d \mathrm{P}_{\mathrm{A}}(t) / d t$
- We can use a $6 \times 6$ matrix to combine these two transformations into a single equation


## Transforming States

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$$
\mathbf{S}_{\mathbf{B}}(t)=\mathbf{T}_{\mathrm{A} \text { to } \mathrm{B}}(t) \mathbf{S}_{\mathrm{A}}(t)
$$

where

$$
\mathbf{S}_{\mathrm{i}}(t)=\left(\begin{array}{l}
\mathbf{P}_{\mathrm{i}}(t)^{d \mathbf{P}_{\mathrm{i}}(t) / d t}
\end{array}\right)_{\mathrm{i}=\mathrm{A} \circ \mathrm{~B} B}
$$

and

$$
\mathbf{T}_{\mathrm{A} \text { to } \mathrm{B}}(t)=\left(\begin{array}{l|l}
\mathbf{R}_{\mathrm{AtoB}}(t) & \mathbf{0} \\
\hline d \mathbf{R}_{\mathrm{AtoB}}(t) / d t & \mathbf{R}_{\mathrm{AtoB}}(t)
\end{array}\right)
$$

The SPICELIB routines SXFORM and PXFORM return state transformation and position transformation matrices respectively.

## Coordinate Systems

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- Planetocentric
- Latitude: measured from X-Y plane
- Longitude: increases counterclockwise w.r.t. the $+Z$ axis
» $+Z$ points to the north side of the invariable plane
- Radius: measured from center of object
- Planetographic, Geodetic, Planetodetic
- Tied to a reference surface
- Latitude: for a point on a reference ellipsoid, angle measured from X-Y plane to the surface normal at the point of interest. For other points, equals latitude at the nearest point on the reference ellipsoid.
- Longitude
» -odetic: same as for planetocentric
" -ographic: longitude of sub-observer point, for a distant, fixed observer in the J2000 frame, increases with time
- Height above reference surface


## Aberration Corrections: Introduction

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- Within the SPICE system, "aberration corrections" are adjustments made to state vectors and timedependent reference frames to accurately reflect the apparent-as opposed to the actual-state and attitude of a target object as seen from a specified observer at a specified time.
- Actual, uncorrected states from an ephemeris are called "geometric" states.
- When computing state vectors, SPICE users may request geometric or aberration-corrected states.
- Aberration corrections are needed to accurately answer questions such as:
- In which direction must a remote sensing instrument be pointed to observe a target of interest?
- For a given pointing direction and observation time, what target body surface location would be observed by a remote sensing instrument?
- In which direction must an antenna be pointed to transmit a signal to a specified target?


## Example: Predicted vs Actual Photo

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We compare the predicted appearance of a photograph from an optical camera against the actual photograph. We show three predictions derived using different aberration corrections: NONE, LT ("light time only"), and LT+S ("light time plus stellar aberration").

For each prediction, we use red overlays to indicate the expected location in the photo of the images of an extended target body (for example, a natural satellite), of features on the surface of the target body, and of a star.


NONE
Predicted images using uncorrected target position and orientation and uncorrected star direction vector


LT
Predicted images using light time-corrected target position and orientation and uncorrected star direction vector


LT+S
Predicted images using light time and stellar aberrationcorrected target position, light time-corrected target orientation, and stellar aberration-corrected star direction vector

## Prediction Without Corrections

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Using geometric target positions, target images in photos or other remote-sensing observations don't appear at their predicted locations.

## Light Time Corrections

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Observer's position at time ET

At time ET, the observer's camera records photons emitted from the target at time ET-LT, where LT is the one-way light time. The camera "sees" the target's position and orientation at ET-LT.

## Prediction Using Light Time Corrections

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Using the light time corrected target position and orientation, the predicted locations in the photo of the target image and surface features have changed, but the accuracy of the prediction may still be poor.

## Stellar Aberration Correction

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Note: The angular separation of the geometric, light time corrected, and apparent positions as seen by the observer has been exaggerated for readability.

Observer's velocity relative to SSB


At time ET, the observer's camera records photons emitted from the target at time ET-LT, where LT is the one-way light time.

The vector from the observer at ET to the location of the target at ET-LT is displaced by a physical phenomenon called stellar aberration. The displaced vector yields the apparent position of the target.

## Prediction Using "LT+S" Corrections

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Using the light time and stellar aberration-corrected target position, light timecorrected target orientation, and stellar aberration-corrected star direction, we obtain a significantly improved image location predictions.
Remaining prediction errors may be due to, among other causes, pointing error, spacecraft and target ephemeris errors, and timing errors.

## Effect of Aberration Corrections - 1

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- Angular offsets between corrected and uncorrected position vectors over the time span 2004 Jan 1--2005 Jan1
- Mars as seen from MEX:
» LT+S vs NONE: . 0002 to .0008 degrees
» LT vs NONE: . 0006 to .0047 degrees
- Earth as seen from MEX:
» LT+S vs NONE: . 0035 to .0106 degrees
» LT vs NONE: . 0000 to .0057 degrees
- MEX as seen from Earth:
» LT+S vs NONE: . 0035 to .0104 degrees
» LT vs NONE: . 0033 to 0048 degrees
- Sun as seen from Mars:
» LT+S vs NONE: . 0042 to .0047 degrees
» LT vs NONE: . 0000 to 0000 degrees


## Effect of Aberration Corrections - 2

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- Angular offsets between corrected and uncorrected position vectors over the time span 2004 Jan 1--2008 Jan1
- Saturn as seen from CASSINI:
» LT+S vs NONE: . 0000 to .0058 degrees
» LT vs NONE: . 0001 to .0019 degrees
- Titan as seen from CASSINI:
» LT+S vs NONE: . 0000 to . 0057 degrees
» LT vs NONE: . 0000 to .0030 degrees
- Earth as seen from CASSINI:
» LT+S vs NONE: . 0000 to .0107 degrees
» LT vs NONE: . 0000 to .0058 degrees
- CASSINI as seen from Earth:
» LT+S vs NONE: . 0000 to .0107 degrees
» LT vs NONE: . 0000 to .0059 degrees
- Sun as seen from CASSINI:
» LT+S vs NONE: . 0000 to .0059 degrees
» LT vs NONE: . 0000 to .0000 degrees

